Belief Revision and Information Economy Principle

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The project

(1) Belief state and new information

(2) Information economy principle and epistemic entrenchment

(3) Rott's counterexample to information economy principle

(4) Preference relation and focal points theory

Belief state and new information

(1) Quine presented the basic idea of information economy principle, or in his words, *principle of minimum mutilation*.

(2) *AGM* theory which proposed by Alchourrón, Gärdenfors and Makinson who attempted to construct a formal account to the principle of information economy.

BELIEF STATE AND NEW INFORMATION

(1) the conclusive function of *AGM* theory is the contraction function

(2) The reason of this thought is that we can transform the revision function thereby the composition of expansion and contraction function by Levi's identity, i.e. $K^{*} = (K^{-}_{-A})^{+}_{A}$

Belief state and new information

The postulates of contraction of *AGM* theory

(K⁻1) For any sentence A and any belief set K, K^-_A is a belief set.

(K⁻2) $K^-_A \subseteq K$。

(K⁻3) If $A \notin K$, then $K^-_A = K$.

(K⁻4) If not $\vdash A$, then $A \notin K^-_A$.

(K⁻5) If $A \in K$, then $K \subseteq (K^-_A)^+_A$.

(K⁻6) If $\vdash A \leftrightarrow B$, then $K^-_A = K^-_B$.

(K⁻7) $K^-_A \cap K^-_B \subseteq K^-_{A\&B}$.

(K⁻8) If $A \notin K^-_{A\&B}$, then $K^-_{A\&B} \subseteq K^-_A$.

Information economy principle and epistemic entrenchment

According the Information Economy Principle, the aim of contraction function is try to retain the most beliefs as possible.

It is meant what the contraction function has done is to create the maximal beliefs which fail to imply the belief, say *A*, which we aims to retract.

Information economy principle and epistemic entrenchment

S is a selection function which picks out the maximal subsets in $K \perp A$ that are ***epistemologically most entrenched.***

(Def Part) $K^{-}_{A} = \cap S(K \perp A)$

Information economy principle and epistemic entrenchment

The key notions of epistemic entrenchment

(i) It is possible to determine the relative epistemic entrenchment of sentences in a belief set *K* independently of what happens to *K* in contractions and revisions.

(ii) When a belief set *K* is contracted (or revised), the sentences in *K* that are given up are those with the *lowest* epistemic entrenchment.

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Information economy principle and epistemic entrenchment

The postulates of epistemic entrenchment

(EE1) For any A, B, and C, if $A \leq B$ and $B \leq C$, then $A \leq C$. (Transitivity)

(EE2) For any A and B, if $A \vdash B$, then $A \leq B$. (Dominance)

(EE3) For any A and B in K, $A \leq A\&B$ or $B \leq A\&B$ (Conjunctiveness)

(EE4) When $K \neq K_{\perp}$, $A \in K$ iff $A \leq B$ for all B. (Minimality)

(EE5) If $B \leq A$ for all B, then A.

Information economy principle and epistemic entrenchment

The significance of the notion of epistemic entrenchment in *AGM* theory due to the contention of the notion of epistemic entrenchment is more fundamental than the contraction or a revision function as they suggested.

Thus we need to give a good account to this notion if we want to construct the whole theory of belief change based on it.

Rott's counterexample

Two dogmas of AGM theory

(1) When accepting a new piece of information, an agent should aim at a minimal change of his old belief.

(2) If there are different ways to effect a belief change, the agent should give up those beliefs which are least entrenched.

ROTT'S COUNTEREXAMPLE

***Observation 1*. No two distinct belief-contravening candidate revisions of a consistent and logically closed belief set by a sentence ϕ can be set-theoretically compared in terms of beliefs on which they differ with the prior belief set.**

***Observation 2*. Suppose we want to revise a belief set by a sentence ϕ and find two elements of the belief set that nonredundantly entail the negation of ϕ. Then it may well be rational, according to the standard belief-revision constructions, to restore consistency by moving the *more* entrenched and retaining the *less* entrenched belief.**

Rott's counterexample

Suppose a belief set K and $\neg\phi$ is in K, and K' and K'' are two distinct revised belief sets of K with respect of ϕ. Let $X \Delta Y$ be the two sets which is symmetric difference defined as $(X \setminus Y) \cup (Y \setminus X)$. If we can show that there is a sentence in $K \Delta K'$ but not in $K \Delta K''$, and vise versa, then the two revision candidate belief set K' and K'' are not related by subset inclusion.

Rott's counterexample

First of all, since $K' \neq K''$ there is either a sentence in $K'-K''$ or there is a sentence in $K''-K'$, let's assume the former that there is a sentence ψ is in K' but not in K''. Next, it is easy to see that the sentence $\neg\phi\vee\psi$ is the logical consequence of $\neg\phi$ and of ψ. It is follows that $\neg\phi\vee\psi$ is in $K \Delta K''$ but not in $K \Delta K'$ since ψ is in K and K' but not in K''. On the other hand, there is another sentence $\phi\wedge\psi$ which is in $K \Delta K'$ but not in $K \Delta K''$ due to $\phi\wedge\psi$ is in K' but not in K and K'', thus the proof is finished.

Rott's counterexample

Suppose there is a belief set *K* which entails ϕ and two distinct sentences ψ and χ which are of the form $(\neg\phi\wedge\alpha)\vee(\phi\wedge\beta)$ and $\neg\phi\vee\neg\beta$ respectively. And assume that the sentence α is less epistemic entrenched than $\neg\phi$, i.e. $\alpha < \neg\phi$, in addition β is in K^*_ϕ.

Rott's counterexample

(P1)

1. $\neg\phi\wedge((\neg\phi\wedge\alpha)\vee(\phi\wedge\beta))$	Premise
2. $(\neg\phi\wedge\alpha)\vee(\phi\wedge\beta)$	1, Simp
3. $\neg\phi$	1, Simp
4. $\neg\phi\vee\neg\beta$	3, Add
5. $\neg(\phi\wedge\beta)$	4, DeM
6. $\neg\phi\wedge\alpha$	2, 5, DS
7. α	6, Simp

Rott's counterexample

(P2) Since $\neg\phi\wedge((\neg\phi\wedge\alpha)\vee(\phi\wedge\beta)) \vdash \alpha$ the result of (P1)

1. $\neg\phi\wedge((\neg\phi\wedge\alpha)\vee(\phi\wedge\beta)) < \alpha$ (EE2, strictly sense)
2. $\neg\phi\wedge\psi < \alpha$ (ψ stands for $((\neg\phi\wedge\alpha)\vee(\phi\wedge\beta))$)
3. $\psi < \alpha$ (because $\alpha < \neg\phi$)
4. $\psi < \neg\phi$ (EE1)
5. $\psi < \neg\phi\vee\neg\beta$ (EE2 & EE1, $\neg\phi\vdash\neg\phi\vee\neg\beta$)
6. $\psi < \chi$ (χ stands for $\neg\phi\vee\neg\beta$)

Response to Rott's counterexample

The strategies in response to Rott's counterexample

(1) Contractions: First the contractions means the contracted belief set should be a subset of the original belief set, so the problem of symmetric difference has vanished. In one word, the revised belief sets K' and K'' of K are either $K' \subset K''$ or $K'' \subset K'$.

(2)&(3) Reconstruction and Dispositions: The second and third ways of defense are to determine the selection function in terms of other non-logical relation or structures, for example, preference relation. As the construction of Grove's sphere model, the relative preference relation is pre-established to make the change of belief set satisfied the requirement of minimization.

Response to Rott's counterexample

The strategies in response to Rott's counterexample

(4) Truths: The last line to response the counterexample is to consider the connection between beliefs and real world as William James said, "We must know the truth; and we must avoid error—these are our first and great commandments as would-be knowers."

Focal points

Suppose you want to meet your friend somewhere in New York but unfortunately you can't communicate to each other at that time. Where is the best area for you to choose? It is surprised that over 70% people chose the same place – Grand Central Station. In this case Grand Center Station is salient for these people who chose it and provides a 'focal point for each person's expectation of what the other expects him to expect to be expected to do.'

Focal points

The first part let us take an account to focal points. Suppose there are two players, 1 and 2 and each player has a finite set of strategies S_i, so the strategies of each player are denoted by s_{1i} and s_{2i}. The strategy pair is denoted by s = (s_{1i}, s_{2j}) and the utility pair is u = (u_1(s), u_2(s)). The main character of focal points theory is the outcome utility is (1, 1) if $i = j$ and (0, 0) otherwise.

Another example

Assume there is a man who always wears his hat when it rains, but when it does not rain, he wears his hat by random. Let *A* be the sentence 'It rained today' and *B* 'The man wears his hat.' Suppose it rained today and I know the man wore his hat because we can derive *B* from *A*, so the sentences *A* and *B* are both in *K*. But when the theory *K* is revised by –*A*, it is obvious that the sentence *B* do not conflict to –*A*. So, it is no need to give up *B* when the sentence *A* has been contracted in this case.

New Gettier's problem

I called that both the justification case mentioned by Gärdenfors and the counterexample which Rott provided are the formal Gettier's style problem. The so-called Gettier problem is the problem of derivation which provided the inappropriate reason to some true beliefs. By the focal points theory insisted, the revision of belief set was no longer a process which is independent to others but a coordinate game to decide which beliefs should be given up or retained.

Thanks for your listening